



LQR optimised control of the AAU³ inverted pendulum



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INTRODUCTION

- ▶ The AAU³ is a reaction controlled inverted pendulum experiment based on ETH Zürich's "Cubli"[1].
- ▶ The inverted pendulum is a classic control problem, frequently used to benchmark control algorithms[2].
- ▶ Using a reaction wheel attached to the pendulum for control is interesting, due to its application in spacecraft attitude control[3].
- ▶ This project aims to develop a robust control system for the AAU³, capable of maintaining an unstable equilibrium while rejecting small external disturbances.

METHODS

- ▶ A linearized third-order model of the system dynamics is derived from the free body diagrams shown in Fig. 2 using Newtonian Mechanics.

$$J_F \ddot{\theta}_F = g \cdot \theta_F \cdot (\ell_F \cdot m_F + m_w \cdot \ell_w) - \ddot{\theta}_F \cdot \ell_w^2 \cdot m_w - b_F \cdot \dot{\theta}_F + b_w \cdot \dot{\theta}_w - \tau_m$$

- ▶ A state space representation of the system is used to construct the block diagram shown in Fig. 3 and analyse the open-loop control properties.
- ▶ The system parameters are obtained through a combination of physical tests and scientific Computer Algebra Systems (CAS) tool estimations.
- ▶ A complementary filter is used to attenuate the accelerometer noise and gyroscope drift.
- ▶ LQR is used to determine the optimal values of the state feedback control matrix[5].

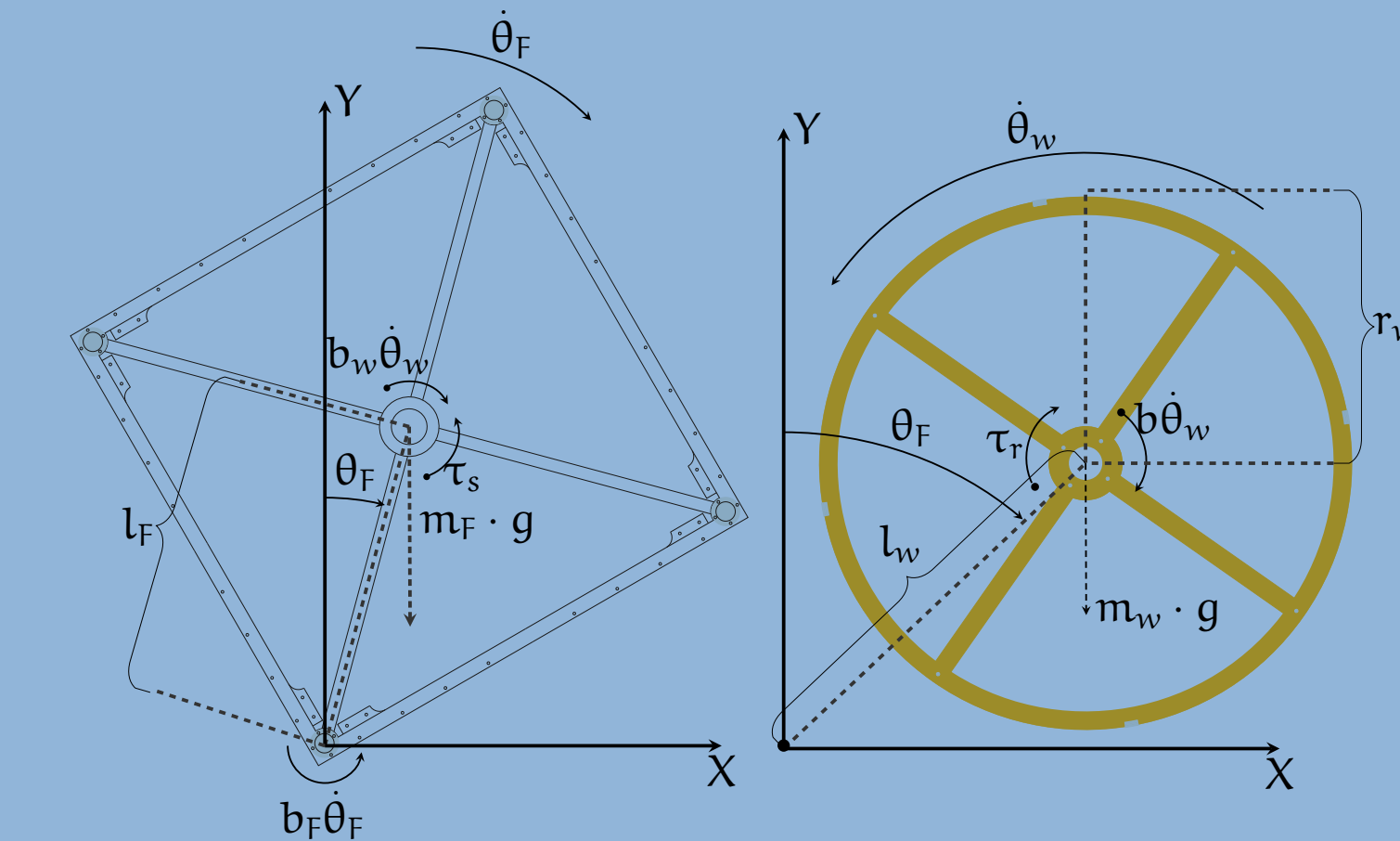


Figure 2: Illustration of system dynamics.

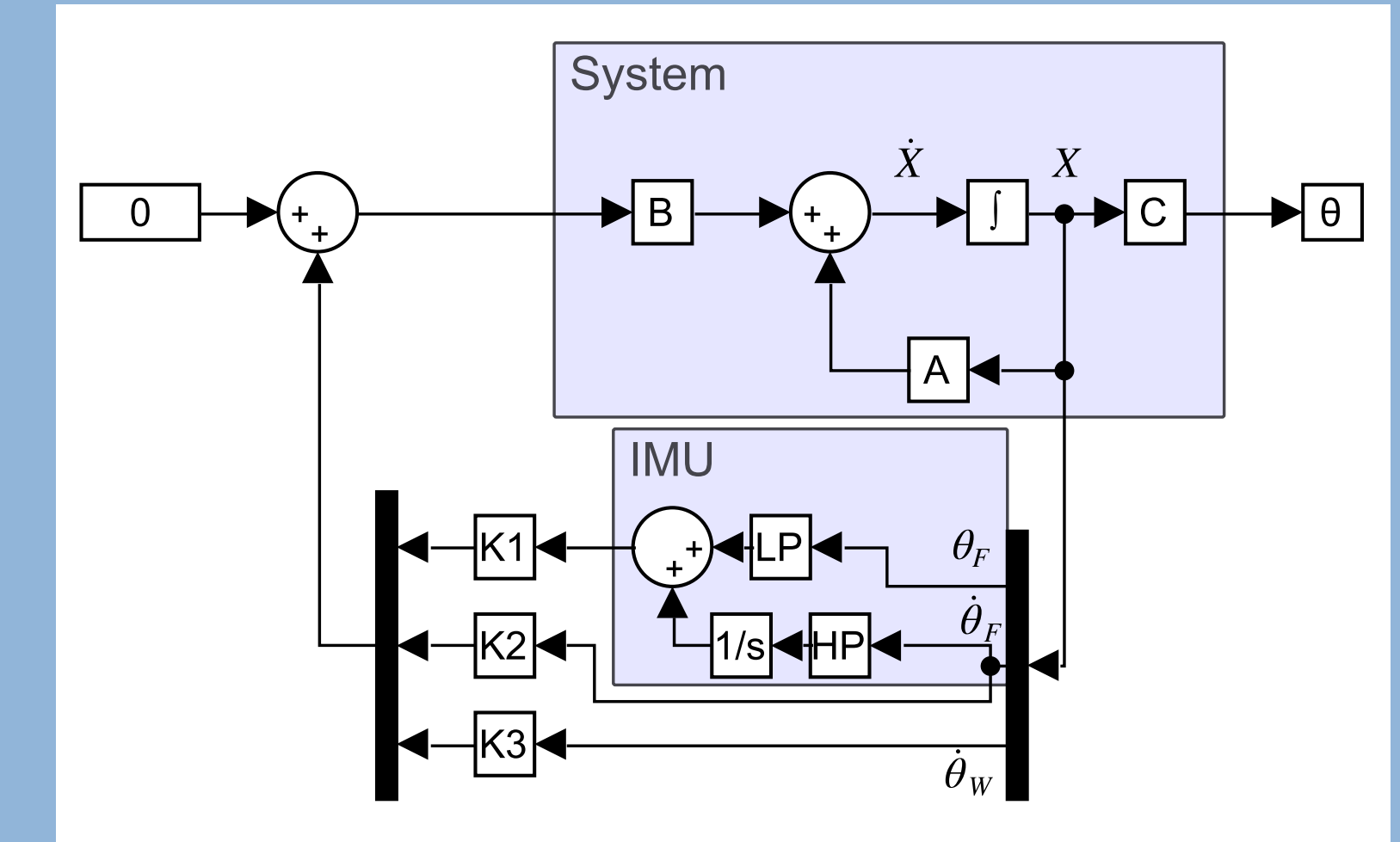
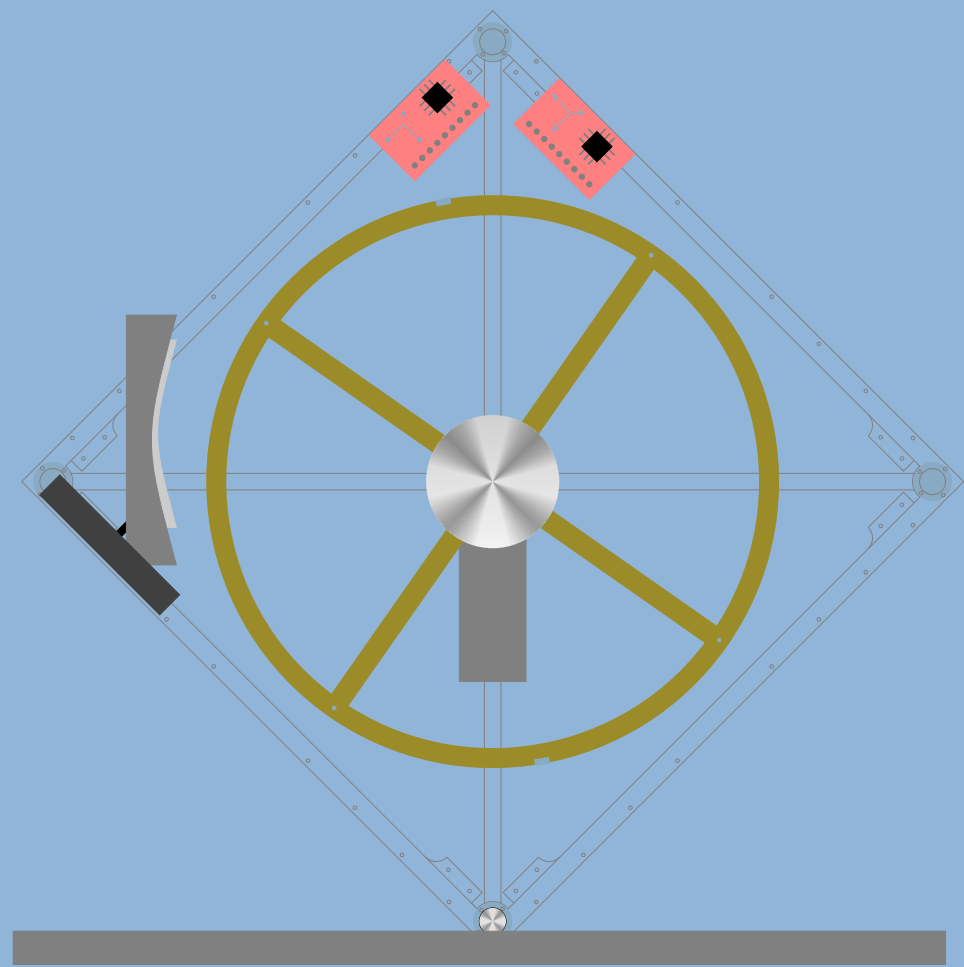


Figure 3: System block diagram.

SYSTEM OVERVIEW

The AAU³, shown in Fig. 1, is an electromechanical system consisting of the components listed below:

Figure 1: AAU³ Diagram

- ▶ A rectangular frame connected to the base plate via a bearing used to mount all other components.
- ▶ A reaction wheel mounted in the centre of the frame via a bearing.
- ▶ An IMU mounted at the top of the frame measures the frame's position and velocity.
- ▶ A brushless DC motor with its rotor and stator mounted to the wheel and frame respectively, is used for actuation.
- ▶ A set of Hall sensors inside the motor housing measures the wheel's velocity.
- ▶ A servo operated friction brake mounted to the side of the frame to rapidly brake the wheel and stand up from a resting position.

RESULTS

- ▶ Fig. 4 compares the performance of the filtered IMU (red graph) to the potentiometer (blue graph). Results show a slower response from the IMU, while maintaining similar accuracy and lower noise.
- ▶ Fig. 5 compares the simulated responses of the LQR controller compared to the pole-placement controller used in the previous prototype [4] and the uncontrolled system. Results show a faster settling time for the LQR with similar overshoot.
- ▶ Fig. 6 shows the acceptance test of the AAU³ balancing at its equilibrium and being pushed to its side and getting up by itself and back to the equilibrium multiple times.
- ▶ The network delay test showed that the AAU³ is stable with a 240 ms impulse delay which corresponds to a packet-loss of 120 packets, though throughout one hour no packet-loss was detected.

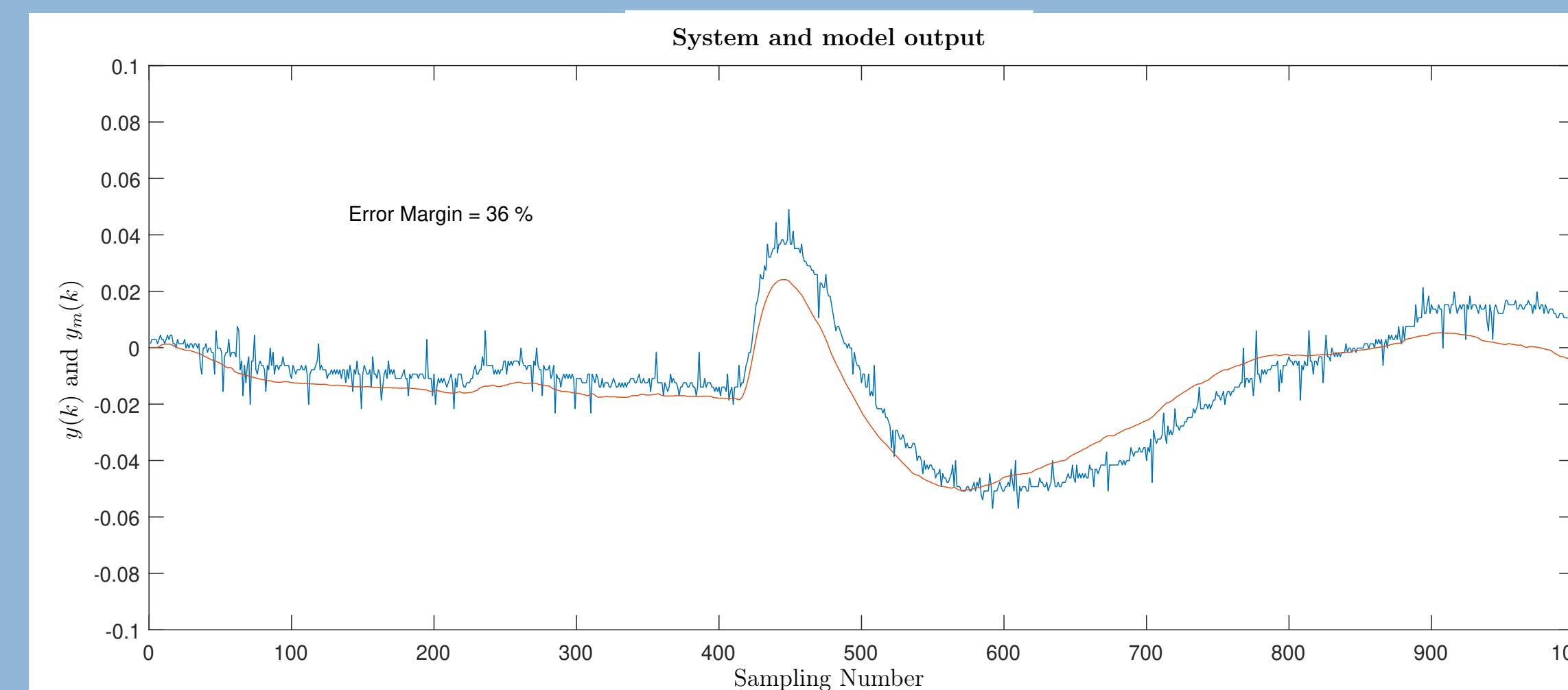


Figure 4: Complementary filter performance.

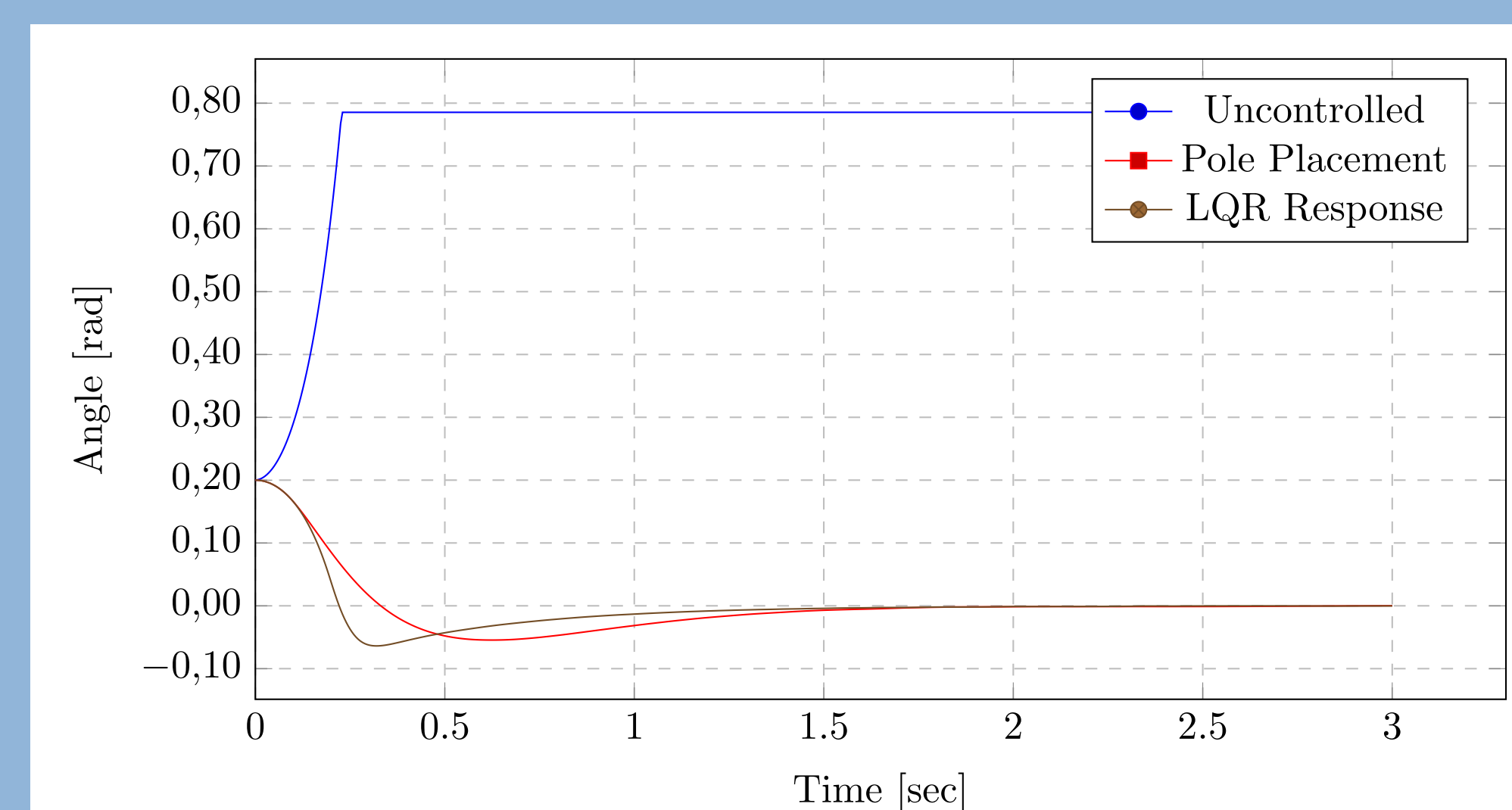


Figure 5: Simulated responses.

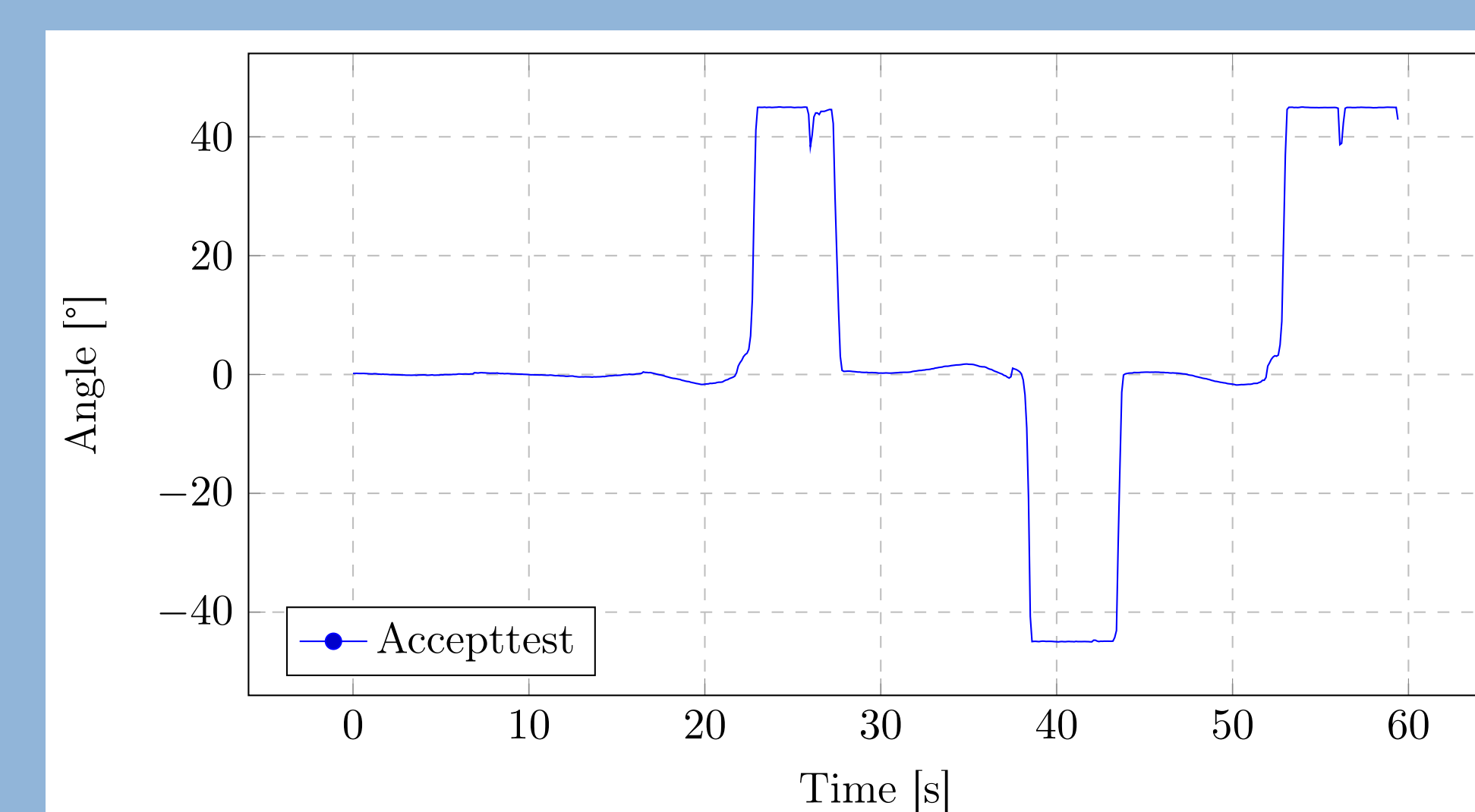


Figure 6: Acceptance test results.

CONCLUSION

The control algorithm proved capable of maintaining stability around the equilibrium point while rejecting disturbances. The control algorithm showed to be an improvement compared to earlier prototypes. The IMU data was filtered with an acceptable margin of error. Furthermore, the AAU³ is capable of jumping up from resting position to its equilibrium.

ACKNOWLEDGEMENTS

The authors of the poster would like to thank Simon Jensen for his sparring with the group regarding past experience with circuit construction for the AAU³.

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